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FUZZY MODEL AND ALGORITHM FOR SOLVING OF E-SHOP INCOME MAXIMIZATION*

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Abstract. In the paper the economic-mathematical model of income maximization using the fuzzy logic is considered for e-trade. The price of the product and other parameters in constraints are considered as the fuzzy parameter and on this basis the fuzzy model of e-shop income maximization is constructed. On the basis of this model the maximization of the income is reduced to the corresponding linear programming problem. To solve this problem the basic idea is to divide the fuzzy numbers on the α-cuts with the definite step. Finally, to illustrate our method, we solve a numerical example for e-shop.

Keywords: mathematical economics, optimization, linear programming, fuzzy numbers, e-trade.

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1. Introduction

One of the most dynamic and perspective market segments is an electronic commerce. Although this trend has a set of unique features, it is based on the general principles of marketing [10]. In electronic commerce, as well as in the usual one, there is a large variety of possible commercial activities. The widespread activities are selling the products and services on the internet. Internet helps businessmen to develop their own business. One of them is e-shop. E-shop is a commercial software specially designed for easy buying and selling goods and services on the websites. In fact, e-shop is the web site, which consists of the specifically oriented e-commerce programs. E-shop has its advantages in comparison with real one:

- optional the available-for-sale goods (but in this case, it needs to have a set of suppliers, working on the "just in time" principle);
- it doesn't need placement for sales;
- sellers' mobility;
- it's possible to expand your business up to the world markets;
- it doesn't need to employ sellers, administrators, managers, cashiers, security guards and someone else;
- active time isn't limited.

But, there are some risks in this kind of business. The risks of e-shop:

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- hacker attack;
- "Bugs" (software errors);
- buying a "pig in a poke";
- clients' mobility.

To take full advantage of the e-shop and to practically reduce to zero above risks, it is necessary to find the really best programmer and the professional software company. This is very important, because with the internet development e-commerce is becoming for businessmen not a luxury but sales promotion devices to broader market [10].

The created e-shop must be functional, convenience, profitable web site and to ensure payback investment for it. In other side internet providers' speed of the users affects to the convenience and profitability of the e-shop. The speed, comfort, security of operations depends on speed and security level of the consumers' provider.

It is clear that in the real and virtual space for the long-term perspective the aim of the company is the profit maximization. The main aim of this study is to develop the adequate economic-mathematical model of income maximization for the e-shop. The price of the product is considered as a fuzzy parameter and on this basis the fuzzy model is developed of e-shop income maximization. On the basis of this model the maximization of the income is reduced to the corresponding linear programming problem. To solve this problem the basic idea is to divide the fuzzy numbers on the α-cuts with the definite step. The paper consists of the sections introduction, problem statement, main results, numerical example for e-shop, conclusion.

2. Problem statement

Let us create the economic-mathematical model and maximize the income of e-shop, using the fuzzy system. To do this, we need to define the basic parameters affecting the income. We first consider the classical (crisp) model of income maximization for the store in the real business:

$$\sum_{i=1}^{n} p_i x_i \to \max \tag{1}$$

$$\sum_{i=1}^{n} a_{ij} x_i \le d_j, j = \overline{1,m} \tag{2}$$

$$x_i \ge 0, i = \overline{1,n}. \tag{3}$$

Here
x_i- volume of the i-th ($i = \overline{1,n}$) product type;
p_i-price of the i-th product type unit;
a_{ij}-j-th kind the costs for per unit of i-th type of product;
d_j- j-th type of costs ($j = \overline{1,m}$).

Notice that the income from the sale of the i-th product in the virtual business depends on the variety of qualitative and quantitative indicators - the price of goods and services, security parameters of the e-shop, payment issues, number of internet users in target country, legislative base of electronic trade, the market for Internet access services, long-term trend of the potential size of the market, seasonal fluctuations in business activity, the quality of Internet service providers, the price attractiveness providers, the influence of the speed of access to Internet resources, sources of the customers' inflow and outflow, and so on., which have the fuzzy descriptions. Since many of these parameters have fuzzy description, taking them into account in the model (1)-(3) as a crisp indications, or are impossible or maybe with the significant assumptions. On the basis of these arguments we can say that the fuzzy description of these parameters in the model may be more adequate than in a sense taking a crisp description.

As we note above the economic-mathematical model of income maximization for the e-shop by using fuzzy logic is investigated in this paper. That is why in the recent studies and publications about e-trade, economic-mathematical modeling, fuzzy logic is analyzed. In [1, 7, 9, 10, 11, 12, 17, 21] was considered internet applications for the various business models and its affects to the profit. In [6] was considered architecture-level software performance abstractions for online performance prediction. The solution methods of fuzzy optimization problems was considered in [2, 3]. The linguistic variables, fuzzy-term sets and fuzzy numbers was considered in detail in [4]. Fuzzy linear programing problems were considered [8, 14, 18]. Fuzzy models of economic systems were considered in [5, 13, 20]. As we know, income of any economic object depends on the price of the products. In [15] the pricefixing problem was studied from the different aspects. In [19] on a homotopy based method for solving systems of linear equations were considered.

The fuzzy models of e-store is not enough studied. This also applies to the fuzzy economic-mathematical model of maximizing the profit of the e-shop that can give very good results.

Let us return to the model (1)-(3). Let us consider the prices of products $p_i, (i = \overline{1,n})$. We know that price is a very aggregate economic parameter. Since it is formulated (image) by influencing the different factors: consumer's choice, fashion, age structure of consumers, consumers' income, the cost of resources, advertising and etc. In e-commerce here adds another technical indicators that we have listed above. Based on these considerations, we take the price of the products as fuzzy parameter.

If the prices of the products -p_i are fuzzy then x_i also be fuzzy. Then $p_i x_i$ hasn't meaning. Then it makes sense to consider $\tilde{p}_i \tilde{x}_i$. In this case, we use the method proposed in [3], in which the space and the scalar product of fuzzy numbers are given. We give some explanations:

Let us assume that F is the class of convex normal fuzzy numbers. For any $a \in F$ the set of α-cuts of fuzzy number a is defined as the interval

$$a^{\alpha} = [L_a(\alpha), R_a(\alpha)]\,, \alpha \in [0,1]\,.$$

Let

$$a = (a_1, a_2)\,, b = (b_1, b_2)\,, a_i, b_i \in F, i = \overline{1,2}$$

be α- cuts of fuzzy number $a_i, b_i \in F$ we denote

$$a_i^{\alpha} = [L_{a_i}(\alpha), R_{a_i}(\alpha)]\,, \alpha \in [0,1]\,,$$

$$b_i^{\alpha} = [L_{b_i}(\alpha), R_{b_i}(\alpha)]\,, \alpha \in [0,1]\,.$$

The scalar product $a \circ b$ in $F \times F$ is defined as follows

$$a \circ b = \frac{1}{2}\int_0^1 \left[(L_{a_1}(\alpha) - L_{a_2}(\alpha))\,(L_{b_1}(\alpha) - L_{b_2}(\alpha)) + (R_{a_1}(\alpha) - R_{a_2}(\alpha))\,(R_{b_1}(\alpha) - R_{b_2}(\alpha))\right] d\alpha$$

Using this approach, instead of (1) we consider the following functional:

$$\frac{1}{2}\int_0^1 \left[L_{p_l}(\alpha) L_{x_l}(\alpha) + R_{p_l}(\alpha) R_{x_l}(\alpha)\right] d\alpha \to \max\,. \tag{4}$$

Here $L_{p_l}(\alpha)$ and $R_{p_l}(\alpha)$ correspondingly, are the left and right boundaries of the α- cuts of the fuzzy number p_i; $L_{x_l}(\alpha)$and $R_{x_l}(\alpha)$ respectively are the left and right boundaries of the $\alpha-$ cuts of the fuzzy number x_i.
The condition (2) will be in the following form:

$$\frac{1}{2}\sum_{i=1}^{n} a_{ij}\int_{0}^{1}\left[L_{x_l}(\alpha)+R_{x_l}(\alpha)\right]d\alpha \leq d_j, j=\overline{1,m}. \tag{5}$$

It is clear that taking into account the fuzziness of , we obtain the following condition:

$$0 \leq L_{x_l}(\alpha) \leq R_{x_l}(\alpha). \tag{6}$$

Note that the functional (4) can be considered as averaged estimate of the function (1).
Let's present an illustrative example.

3. Main results

Problem (4)-(6) is not a linear programming problem. Here the variables $\left[L_{x_l}(\alpha) \leq R_{x_l}(\alpha)\right], \alpha \in [0,1]$, confirm that (4) is functional. For correspondence this functional to the linear programming problem, the integrals in (4) and (5) should be discretized at the α- cuts. Let's divide the interval [0, 1] into N parts with step h ($h=\frac{1}{N}$) Then the problem (4)-(6) will be in the following form:

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{N}\left[L_{p_l}(\alpha_k)L_{x_l}(\alpha_k)+R_{p_l}(\alpha_k)R_{x_l}(\alpha_k)\right] \to \max. \tag{7}$$

$$\frac{1}{2}\sum_{i=1}^{n} a_{ij}\sum_{k=1}^{N}\left[L_{x_l}(\alpha_k)+R_{x_l}(\alpha_k)\right] \leq d_j, j=\overline{1,m}. \tag{8}$$

$$0 \leq L_{x_l}(\alpha_k) \leq R_{x_l}(\alpha_k) \leq 1. \tag{9}$$

Here $l_{ik}=L_{x_l}(\alpha_k), r_{ik}=R_{x_l}(\alpha_k), i=\overline{1,n}, k=\overline{1,N}$.
If the parameters a_{ij} are fuzzy, then the model (7)-(9) will be in the following form:

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{N}\left[L_{p_l}(\alpha_k)L_{x_l}(\alpha_k)+R_{p_l}(\alpha_k)R_{x_l}(\alpha_k)\right] \to \max. \tag{10}$$

$$\frac{1}{2N}\sum_{i=1}^{n}\sum_{k=1}^{N}\left[L_{a_{lj}}(\alpha_k)L_{x_l}(\alpha_k)+R_{a_{lj}}(\alpha_k)R_{x_l}(\alpha_k)\right] \leq d_j, j=\overline{1,m}. \tag{11}$$

$$0 \leq L_{x_l}(\alpha_k) \leq R_{x_l}(\alpha_k) \leq 1. \tag{12}$$

As we see, in comparison with the classical problem the number of initial variables is increased. In the classical problem the number of variables was n, but here n $\times$ 2N. At the same time, the number of conditions is also increased.
In the general these types of problems are solved using the programs packet MATLAB. We suggest the following algorithm:
Step 1. p_i and a_{ij} fuzzy coefficients are given by α- cuts $\alpha \in [0;1]$ in the following form: $p_i^{\alpha}=[u_i+c_i\alpha; m_i-y_i\alpha]$ and $a_{i_i}^{\alpha}=[w_{ij}+v_{ij}\alpha; g_{ij}-q_{ij}\alpha], j=\overline{1,m}, i=\overline{1,n}$ i.e. $L_{p_i}^{\alpha}=u_i+c_i\alpha$, $R_{p_i}^{\alpha}=m_i-y_i\alpha$ $L_{d_j}^{\alpha}=w_j+v_j\alpha$, $R_{d_j}^{\alpha}=g_j-q_j\alpha$.

Step 2. Take the initial step $h_k = \eta$ at $k = 0$, where η is a natural number. From $\alpha(l) = 0$ to $\alpha(l) := \alpha(l) + \frac{1}{h_k}$ calculate $p_i^{\alpha(l)} = [u_i + c_i\alpha(l); m_i - y_i\alpha(l)]$ and $a_{i_i}^{\alpha} = [w_{ij} + v_{ij}\alpha; g_{ij} - q_{ij}\alpha]$, $j = \overline{1,m}, i = \overline{1,n}$, i.e. $L_{p_i}^{\alpha} = u_i + c_i\alpha$, $R_{p_i}^{\alpha} = m_i - y_i\alpha$ $L_{d_j}^{\alpha} = w_j + v_j\alpha$, $R_{d_j}^{\alpha} = g_j - q_j\alpha$.
Step 3. Substituting these values in the problem (10)-(12) we find the solutions $L_{x_i}^{\alpha(l)}$ and $R_{x_i}^{\alpha(l)}$, $i = \overline{1,n}$, $l = \overline{0,h_k}$.
Step 4. On the basis of the solutions we calculate the value of the functional (10) and denote it by $J(h_k)$.
Step 5. Define $h_k := h_k + \Delta h$ at $k := k + 1$
Step 6. If $J(h_{k+1}) \geq J(h_k)$ is satisfied then go to Step 2, else the process is finished. We take as solution the values of the variables of latest $J(h_{k+1})$ which satisfies condition $J(h_{k+1}) \geq J(h_k)$.
Let's explain some details on Step 1: here $u_i, c_i, m_i, y_i, w_{ij}, v_{ij}, g_{ij}, q_{ij}$ ($j = \overline{1,m}, i = \overline{1,n}$)-are given constants.
Let present the example.

4. Numerical example for e-shop

Let $\tilde{x}_1, \tilde{x}_2$ are products, p_1^{α}, p_2^{α}- corresponding prices of products:

$$p_1^{\alpha} = [2 + 2\alpha, 6 - 2\alpha],$$

$$p_2^{\alpha} = [1 + \alpha, 3 - \alpha].$$

A is the matrix of costs norm, each element of this matrix a_{ij} -is the j-th type of unit costs of i-th type of product:

$$A = \begin{bmatrix} \tilde{1} & \tilde{3} \\ \tilde{2} & \tilde{5} \end{bmatrix}, \alpha \in [0,1]$$

d-is the vector of costs, each element of this vector d_j -volume of the j-th type of costs:

$$d = \begin{bmatrix} 4 \\ 6 \end{bmatrix}.$$

Here $\tilde{1} = [\alpha, 2 - \alpha], \tilde{3} = [2 + \alpha, 4 - \alpha], \tilde{2} = [1 + \alpha, 3 - \alpha], \tilde{5} = [4 + \alpha, 6 - \alpha]$., $\alpha \in [0,1]$.
Let's divide parts the interval[0;1] into 10 with step h ($h = \frac{1}{10}$) Then the problem (10)-(12) will be in the following form:

$$\frac{1}{2*11}\sum_{i=1}^{n}\sum_{k=0}^{10}[L_{p_l}(\alpha_k)L_{x_l}(\alpha_k) + R_{p_l}(\alpha_k)R_{x_l}(\alpha_k)] \to \max. \tag{13}$$

$$\frac{1}{2*11}\sum_{i=1}^{n}\sum_{k=0}^{10}\left[L_{a_{lj}}(\alpha_k)L_{x_l}(\alpha_k) + R_{a_{lj}}(\alpha_k)R_{x_l}(\alpha_k)\right] \leq d_j, j = \overline{1,2}. \tag{14}$$

$$0 \leq L_{x_l}(\alpha_k) \leq R_{x_l}(\alpha_k) \leq 1. \tag{15}$$

Here the initial $l_{ik} = L_{x_l}(\alpha_k), r_{ik} = R_{x_l}(\alpha_k), i = \overline{1,2}, k = \overline{0,10}$.
This problem was solving using package MATLAB and obtained results are given in the Table 1.

Table 1.

Fmax	H
14.6636	0.0010
14.7629	0.0020
15.0101	0.0030
15.3858	0.0040
15.8896	0.0050
16.5435	0.0060
17.3771	0.0070
18.3991	0.0080
19.7301	0.0090
21.4275	0.0100
23.8814	0.0110
26.8810	0.0120
32.1210	0.0130
40.0969	0.0140
54.1646	0.0150

Here are given the values of fuzzy x_1, x_2 for each α-cut, $\alpha \in [0; 1]$ with the step h=0,15. As we see, $x_2 = 0$, but $x_1 \neq 0$. The graphic x_1 is shown at the Fig.1.

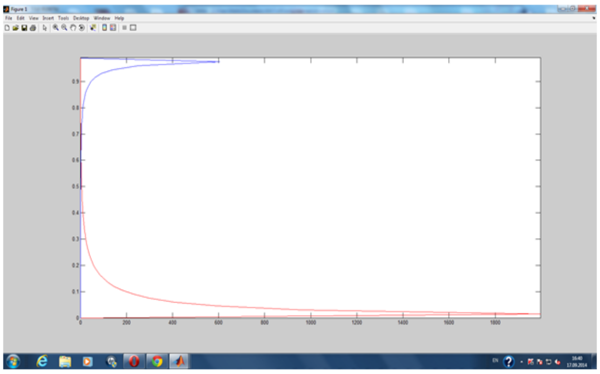

Figure 1. The crisp value of the functional equals to 1.199999999999998e+001.

Let's consider the value of functional on each α-cut, $\alpha \in [0; 1]$ of step h=0.015. Graphically the functional will be as follows (see Fig.2.)

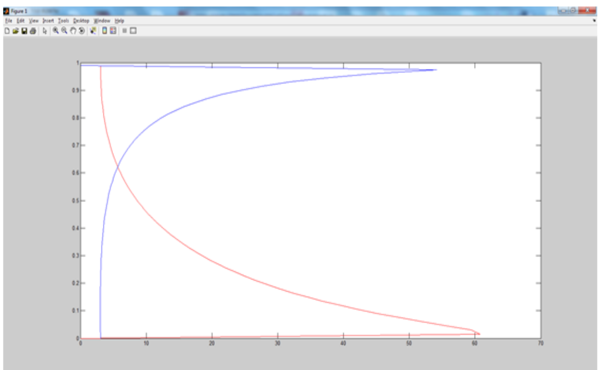

Figure 2. As we see from figure at $\alpha \to 1$ the functional value approximates to 54.1646.

5. Conclusions

In this paper we have proposed a resolution method for a linear programming problem with fuzzy parameters. Through this idea of feasible optimal solution in all α-cuts, the decision-maker has enough information to fix an aspiration level. In other works about fuzzy linear programming problems, fuzzy parameters are defuzzyficated and the problem reduced to the ordinary linear programming problem or reduced to the parametric linear programming problem. But in this paper using all fuzzy information we find fuzzy solution of the problem. It gives the decision-maker to see minimum and maximum levels of realization his (her) wishes depending on solution in all α-cuts. It's very important for the decision-maker in all real economic problems.

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